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Here are my thoughts on AFP loss correction:

We make a series of polarization measurements as the target spins down. Each value measured is lower by $1 - \delta$ than the actual polarization because of AFP losses. Here, δ represents the AFP loss per measurement. We will call the first *measured* polarization P_{m_1} , the second *measured* polarization P_{m_2} , etc. P_{m_2} is lower than P_{m_1} due to actual polarization loss, which depends on the *true* cell lifetime Γ_t . P_{m_2} is further reduced by the AFP loss associated with the measurement. The following expression relates the two measurements,

$$P_{m_2} = P_{m_1} e^{-\Gamma_t \Delta t_2} (1 - \delta)$$

where Δt_2 is the time elapsed between measurements P_{m_1} and P_{m_2} . A similar relation between P_{m_2} and the third measurement P_{m_3} can be written,

$$P_{m_3} = P_{m_2} e^{-\Gamma_t \Delta t_3} (1 - \delta)$$

where Δt_3 is the time elapsed between measurements P_{m_2} and P_{m_3} . Combining these two equations we get,

$$P_{m_3} = P_{m_1} e^{-\Gamma_t (\Delta t_2 + \Delta t_3)} (1 - \delta)^2$$

So for the n^{th} measurement we can write,

$$P_{m_n} = P_{m_1} e^{-\Gamma_t \Delta t} (1 - \delta)^{n-1}$$

where Δt is the total time elapsed between measurements P_{m_1} and P_{m_n} .

The term $(1 - \delta)^{n-1}$ can be expressed as an exponential as follows,

$$e^{-a} = (1 - \delta)^{n-1}$$

where

$$a = -(n - 1) \ln(1 - \delta)$$

Then,

$$P_{m_n} = P_{m_1} e^{-(a/\Delta t + \Gamma_t)\Delta t}$$

or alternatively,

$$P_{m_n} = P_{m_1} e^{-\Gamma_m \Delta t} \tag{1}$$

where

$$\Gamma_m = \Gamma_t - a/\Delta t$$

This means that there is an exponential curve which fits the first and n^{th} *measured* point. Note that Γ_m depends on the total time that has elapsed and the number of measurements made during that time. The point here is that equation 1 is a curve which fits between the first and n^{th} data point. It is not appropriate to use it to fit all measured points to extract Γ_m because Γ_m is not a constant for the spin down curve.

There is one case where this expression works, namely when the time between measurements is constant, say Δt . Then we can write

$$P_{m_n} = P_{m_1} e^{-(a/\Delta t + \Gamma_t)\Delta t}$$

and thus

$$P_{m_n} = P_{m_1} e^{-(n-1)[\ln(1-\delta)/\Delta t + \Gamma_t]\Delta t}$$

and

$$P_{m_n} = P_{m_1} e^{-(n-1)\Gamma_m \Delta t}$$

where $\Gamma_m = \ln(1 - \delta)/\Delta t + \Gamma_t$ is a constant and the curve is fit as a function of n .